

Laugfristige HA - Gleichungen + Ungleichungen

1. a) $7x - 6 = 8x - 5 \quad | -7x + 5$
 $\underline{\underline{-1 = x}}$

Pr: $-7 - 6 = -8 - 5$
 $-13 = -13$

b) $7 - 8x = 6x - 35 \quad | -6x - 7$
 $-14x = -42 \quad | :(-14)$
 $\underline{\underline{x = 3}}$

Pr: $-17 = -17$

c) $7 + 5x = 16 + 8x \quad | -8x - 7$
 $-3x = 9 \quad | :(-3)$
 $\underline{\underline{x = -3}}$

Pr: $-8 = -8$

2. a) $9(9x - 9) + 8x = 542$
 $81x - 81 + 8x = 542 \quad | +81$
 $89x = 623 \quad | :89$
 $\underline{\underline{x = 7}}$

Pr: $542 = 542$

b) $7(4x - 1) - 4x = -103$
 $28x - 7 - 4x = -103 \quad | +7$
 $24x = -96 \quad | :24$
 $\underline{\underline{x = -4}}$

Pr: $-103 = -103$

c) $2(-6x - 7) = 161 - (7 - 9x)$
 $-12x - 14 = 161 - 7 + 9x$
 $-12x - 14 = 154 + 9x \quad | -9x + 14$
 $-21x = 168 \quad | :(-21)$
 $\underline{\underline{x = -8}}$

Pr: $82 = 82$

d) $-6(4 + 4x) = 84 - 3(6 + 2x)$
 $-24 - 24x = 84 - 18 - 6x$
 $-24 - 24x = 66 - 6x \quad | +6x + 24$
 $-18x = 90 \quad | :(-18)$
 $\underline{\underline{x = -5}}$

Pr: $96 = 96$

e) $-(4 + 7x) = 5(4 + x) - 48$
 $-4 - 7x = 20 + 5x - 48$
 $-4 - 7x = 5x - 28 \quad | -5x + 4$
 $-12x = -24 \quad | :(-12)$
 $\underline{\underline{x = 2}}$

Pr: $-18 = -18$

3. a) $13x + 5(2 - 3x) = 8(3x - 2) - 7x$
 $13x + 10 - 15x = 24x - 16 - 7x$
 $-2x + 10 = 17x - 16 \quad | -17x + 10$
 $-19x = -26 \quad | :(-19)$
 $\underline{\underline{x = \frac{26}{19}}}$

Pr: $7\frac{5}{19} = 7\frac{5}{19}$

b) $8(2x - 5) + 3(2x + 3) = 6(8 - x) - 2x + 71$
 $16x - 40 + 6x + 9 = 48 - 6x - 2x + 71$
 $22x - 31 = -8x + 119 \quad | +8x + 31$
 $30x = 150 \quad | :30$
 $\underline{\underline{x = 5}}$

Pr: $79 = 79$

c) $5(3x - 7) + 4(6 - x) = 4 + 3(x + 20) - 2x$
 $15x - 35 + 24 - 4x = 4 + 3x + 60 - 2x$
 $11x - 11 = x + 64 \quad | -x + 11$
 $10x = 75 \quad | :10$
 $\underline{\underline{x = 7,5}}$

Pr: $71,5 = 71,5$

$$\begin{aligned}
 4. \quad a) \quad (x+4)(x+3) &= (x+1)(x+7) \\
 x^2+3x+4x+12 &= x^2+7x+x+7 \\
 x^2+7x+12 &= x^2+8x+7 \quad | -x^2 \\
 7x+12 &= 8x+7 \quad | -7x-7 \\
 \underline{5} &= x \quad \text{Pr: } 72=72
 \end{aligned}$$

$$\begin{aligned}
 b) \quad (x+11)(x-9) &= (x+3)(x-7) \\
 x^2-9x+11x-99 &= x^2-7x+3x-21 \\
 x^2+2x-99 &= x^2-4x-21 \quad | -x^2 \\
 2x-99 &= -4x-21 \quad | +4x+99 \\
 6x &= 78 \quad | :6 \\
 \underline{x} &= \underline{13} \quad \text{Pr: } 96=96
 \end{aligned}$$

$$\begin{aligned}
 c) \quad (x+4)(x-9)-13 &= (x-5)(x-7) \\
 x^2-9x+4x-36-13 &= x^2-7x-5x+35 \\
 x^2-5x-49 &= x^2-12x+35 \quad | -x^2 \\
 -5x-49 &= -12x+35 \quad | +12x+49 \\
 7x &= 84 \quad | :7 \\
 \underline{x} &= \underline{12} \quad \text{Pr: } 35=35
 \end{aligned}$$

$$\begin{aligned}
 d) \quad (3x-5)(2x+5) &= (x+1)(6x-4) \\
 6x^2+15x-10x-25 &= 6x^2-4x+6x-4 \\
 6x^2+5x-25 &= 6x^2+2x-4 \quad | -6x^2 \\
 5x-25 &= 2x-4 \quad | -2x+25 \\
 3x &= 21 \quad | :3 \\
 \underline{x} &= \underline{7} \quad \text{Pr: } 304=304
 \end{aligned}$$

$$\begin{aligned}
 5. \quad a) \quad (x+3)^2 &= (x+2)^2 + 15 \\
 (x+3)(x+3) &= (x+2)(x+2) + 15 \\
 x^2+6x+9 &= x^2+4x+4+15 \quad | -x^2 \\
 x^2+6x+9 &= x^2+4x+19 \quad | -x^2 \\
 6x+9 &= 4x+19 \quad | -4x-9 \\
 2x &= 10 \quad | :2 \\
 \underline{x} &= \underline{5} \quad \text{Pr: } 64=64
 \end{aligned}$$

$$\begin{aligned}
 b) \quad (x+3)^2 + (x-2)^2 &= (x+4)^2 + (x-5)^2 \\
 x^2+6x+9+x^2-4x+4 &= x^2+8x+16+x^2-10x+25 \\
 2x^2+2x+13 &= 2x^2-2x+41 \quad | -2x^2 \\
 2x+13 &= -2x+41 \quad | +2x-13 \\
 4x &= 28 \quad | :4 \\
 \underline{x} &= \underline{7} \quad \text{Pr: } 125=125
 \end{aligned}$$

$$\begin{aligned}
 c) \quad (x-5)^2 + (x-9)^2 &= (x-2)^2 + (x-3)^2 + 3 \\
 x^2-10x+25+x^2-18x+81 &= x^2-4x+4+x^2-6x+9+3 \\
 2x^2-28x+106 &= 2x^2-10x+16 \quad | -2x^2 \\
 -28x+106 &= -10x+16 \quad | +10x-106 \\
 -18x &= -90 \quad | :(-18) \\
 \underline{x} &= \underline{5} \quad \text{Pr: } 16=16
 \end{aligned}$$

$$6. a) \frac{1}{4}x + \frac{2}{3}x + 14 = \frac{1}{3}x + 2 - \frac{5}{12}x \quad | \cdot 12$$

$$3x + 8x + 168 = 4x + 24 - 5x$$

$$11x + 168 = -x + 24$$

$$12x = -144$$

$$\underline{x = -12}$$

$$1x - 168$$

$$| : 12$$

$$\text{Pr: } 3 = 3$$

$$b) \frac{x}{5} + \frac{2x}{10} + 15 = \frac{10x}{5} + \frac{1}{5} + 10 \quad | \cdot 10$$

$$2x + 2x + 150 = 20x + 2 + 100$$

$$4x + 150 = 20x + 102$$

$$-16x = -48$$

$$\underline{x = 3}$$

$$| -20x -150$$

$$| : (-16)$$

$$\text{Pr: } 16\frac{1}{5} = 16\frac{1}{5}$$

$$c) 12\frac{3}{5}x - 11 = 31,6 + \frac{1}{20}x - 17,5 \quad | \cdot 20$$

$$252x - 220 = 632 + x - 350$$

$$252x - 220 = x + 282$$

$$251x = 502$$

$$\underline{x = 2}$$

$$| -x + 220$$

$$| : 251$$

$$\text{Pr: } 14\frac{1}{5} = 14\frac{1}{5}$$

$$d) \frac{2}{3}x + 0,7 = \frac{1}{2}x - 0,05 + \frac{1}{8}x \quad | \cdot 24$$

$$16x + 16,8 = 12x - 1,2 + 3x$$

$$16x + 16,8 = 15x - 1,2$$

$$\underline{x = -18}$$

$$| -15x - 16,8$$

$$\text{Pr: } -11,3 = -11,3$$

$$7. a) \frac{7x}{2} - \frac{3x}{4} = \frac{11}{4} \quad | \cdot 4$$

$$14x - 3x = 11$$

$$11x = 11$$

$$\underline{x = 1}$$

$$\text{Pr: } \frac{11}{4} = \frac{11}{4}$$

$$b) \frac{3x}{5} - \frac{1}{15} = \frac{2x}{3} \quad | \cdot 15$$

$$9x - 1 = 10x$$

$$-1 = x$$

$$\text{Pr: } -\frac{2}{3} = -\frac{2}{3}$$

$$c) \frac{10}{x} = \frac{25}{x} - 3 \quad | \cdot x$$

$$10 = 25 - 3x \quad | -25$$

$$-15 = -3x \quad | : (-3)$$

$$\underline{x = 5}$$

$$\text{Pr: } 2 = 2$$

$$d) \frac{20}{x} = \frac{5}{x} + 3 \quad | \cdot x$$

$$20 = 5 + 3x \quad | -5$$

$$15 = 3x \quad | : 3$$

$$\underline{x = 5}$$

$$\text{Pr: } 4 = 4$$

$$e) \frac{x-3}{x+3} = \frac{x-5}{x-2}$$

$$\begin{aligned} (x-3)(x-2) &= (x-5)(x+3) \\ x^2 - 2x - 3x + 6 &= x^2 + 3x - 5x - 15 \\ x^2 - 5x + 6 &= x^2 - 2x - 15 \quad | -x^2 \\ -5x + 6 &= -2x - 15 \quad | +2x + 6 \\ -3x &= -21 \quad | :(-3) \\ x &= 7 \end{aligned}$$

$$\text{Pr: } \frac{2}{5} = \frac{2}{5}$$

$$f) \frac{x+4}{x-2} = \frac{x+6}{x-1}$$

$$\begin{aligned} (x+4)(x-1) &= (x+6)(x-2) \\ x^2 - x + 4x - 4 &= x^2 - 2x + 6x - 12 \\ x^2 + 3x - 4 &= x^2 + 4x - 12 \quad | -x^2 \\ 3x - 4 &= 4x - 12 \quad | -3x + 12 \\ 8 &= x \end{aligned}$$

$$\text{Pr: } 2 = 2$$

$$8. a) \begin{aligned} 8x - 5x &= 72 \\ 3x &= 72 \\ x &= 24 \end{aligned}$$

Die Zahl heißt 24.

$$\begin{aligned} b) \begin{aligned} 1. \text{ Zahl: } & x \\ 2. \text{ Zahl: } & 3x + 4 \\ 3. \text{ Zahl: } & 5x - 12 \end{aligned} \end{aligned}$$

$$\begin{aligned} x + 3x + 4 + 5x - 12 &= 154 \\ 9x - 8 &= 154 \\ 9x &= 162 \\ x &= 18 \end{aligned}$$

Die Zahlen sind 18, 58 und 78.

$$\begin{aligned} c) \begin{aligned} x + x + 1 + x + 2 + x + 3 &= 178 \\ 4x + 6 &= 178 \\ 4x &= 172 \\ x &= 43 \end{aligned} \end{aligned}$$

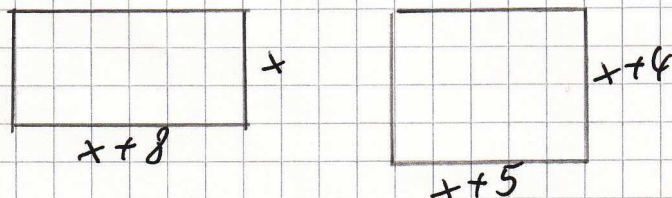
Die Zahlen sind 43, 44, 45 und 46.

$$\begin{aligned} d) \begin{aligned} 1. \text{ Preis: } & x + 72 \\ 2. \text{ Preis: } & x + 36 \\ 3. \text{ Preis: } & x \end{aligned} \end{aligned}$$

$$\begin{aligned} x + 72 + x + 36 + x &= 360 \\ 3x + 108 &= 360 \\ 3x &= 252 \\ x &= 84 \end{aligned}$$

Der erste Preis erhält 156 €, der zweite Preis 120 € und der dritte Preis 84 €.

e)

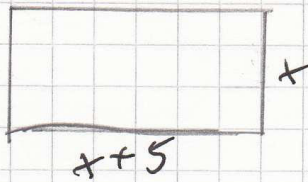


$$\begin{aligned} x(x+8) + 26 &= (x+5)(x+4) \\ x^2 + 8x + 26 &= x^2 + 4x + 5x + 20 \\ x^2 + 8x + 26 &= x^2 + 9x + 20 \quad | -x^2 \\ 8x + 26 &= 9x + 20 \quad | -8x - 20 \\ 6 &= x \end{aligned}$$

ursprüngliches Rechteck: 6 cm und 14 cm $\rightarrow A = 84 \text{ cm}^2$

neues Rechteck: 10 cm und 11 cm $\rightarrow A = 110 \text{ cm}^2$
 $\rightarrow \text{Differenz } 26 \text{ cm}^2$

f)



$$\begin{aligned}
 x+x+5+x+x+5 &= 34 \\
 4x+10 &= 34 \\
 4x &= 24 \\
 x &= 6
 \end{aligned}$$

Länge: 11 cm Breite: 6 cm

Flächeninhalt: 66 cm^2

9. a) $17y + 11 < 16y - 3 \quad (y \in \mathbb{Q})$

$$\begin{aligned}
 & \quad \quad \quad | -16y \\
 y + 11 & < -3 \\
 & \quad \quad \quad | -11 \\
 y & < -14 \\
 L &= \{y \in \mathbb{Q}, y < -14\}
 \end{aligned}$$

b) $11z - 3 < 10z + 4 \quad (z \in \mathbb{N})$

$$\begin{aligned}
 & \quad \quad \quad | -10z \\
 z - 3 & < 4 \\
 & \quad \quad \quad | +3 \\
 z & < 7 \\
 L &= \{0; 1; 2; 3; 4; 5; 6\}
 \end{aligned}$$

c) $0,1x - 13 \leq -10 - 0,9x \quad (x \in \mathbb{N})$

$$\begin{aligned}
 & \quad \quad \quad | +0,9x \\
 x - 13 & \leq -10 \\
 & \quad \quad \quad | +13 \\
 x & \leq 3 \\
 L &= \{0; 1; 2; 3\}
 \end{aligned}$$

d) $34x - 7 > 24 + 33x - 25 \quad (x \in \mathbb{Q})$

$$\begin{aligned}
 & \quad \quad \quad | -33x \\
 34x - 7 & > 33x - 1 \\
 & \quad \quad \quad | -8 \\
 x - 7 & > -1 \\
 & \quad \quad \quad | +8 \\
 x & > 7 \\
 L &= \{x \in \mathbb{Q} ; x > 7\}
 \end{aligned}$$

e) $0,8y - 0,8 + 3,6y \leq 2,4 + 3,4y - 1,7 \quad (y \in \mathbb{N})$

$$\begin{aligned}
 & \quad \quad \quad | -3,4y \\
 4,4y - 0,8 & \leq 3,4y + 0,7 \\
 & \quad \quad \quad | -0,8 \\
 y - 0,8 & \leq 0,7 \\
 & \quad \quad \quad | +0,8 \\
 y & \leq 1,5 \\
 L &= \{0; 1\}
 \end{aligned}$$

$$f) (1,77x + 11) - (8 + x) > (x - 4) + (1,77x + 11) \quad (x \in \mathbb{Q}_+)$$

$$1,77x + 11 - 8 - x > x - 4 + 1,77x + 11$$

$$-6,23x + 7 > 2,77x + 7 \quad | -2,77x$$

$$-9x + 7 > 7 \quad | -7$$

$$-9x > 0 \quad | : (-9)$$

$$x > 0 \quad \underline{L = \{x \in \mathbb{Q}_+; x > 0\}}$$

$$10. a) 2x - (8 - x) < 8(2x + 3) - 5x \quad (x \in \mathbb{R})$$

$$2x - 8 + x < 16x + 24 - 5x$$

$$3x - 8 < 11x + 24 \quad | -11x$$

$$-8x - 8 < 24 \quad | +8$$

$$-8x < 32 \quad | : (-8)$$

$$x > -4$$

$$\underline{L = \{x \in \mathbb{R}, x > -4\}}$$

$$b) 3; 0; \frac{1}{2}; 5,2$$

$$11. a) 15 - 3x > 5(x - 1) \quad (x \in \mathbb{Q}_+)$$

$$15 - 3x > 5x - 5 \quad | +3x$$

$$15 > 8x - 5 \quad | +5$$

$$20 > 8x \quad | : 8$$

$$2,5 > x$$

$$x < 2,5$$

$$L = \{x \in \mathbb{Q}_+; x < 2,5\}$$

$$b) 0,05$$

$$12. a) V = \frac{1}{3} \pi r^2 h \quad | \cdot 3$$

$$3V = \pi r^2 h \quad | : \pi r^2$$

$$\underline{h = \frac{3V}{\pi r^2}}$$

$$b) V = \frac{4}{3} \pi r^3 \quad | \cdot 3$$

$$3V = 4\pi r^3 \quad | : 4\pi$$

$$\frac{3V}{4\pi} = r^3 \quad | \sqrt[3]{}$$

$$\underline{r = \sqrt[3]{\frac{3V}{4\pi}}}$$

$$c) A_0 = \pi r(r+s) \quad | : \pi r$$

$$\frac{A_0}{\pi r} = r+s \quad | -r$$

$$\underline{\underline{s = \frac{A_0}{\pi r} - r}}$$

$$d) A = \frac{a+c}{2} \cdot h \quad | \cdot 2$$

$$2A = (a+c) \cdot h \quad | : h$$

$$\frac{2A}{h} = a+c \quad | -c$$

$$\underline{\underline{a = \frac{2A}{h} - c}}$$

$$e) R = S \cdot \frac{l}{A} \quad | \cdot A$$

$$R \cdot A = S \cdot l \quad | : R$$

$$\underline{\underline{A = \frac{S \cdot l}{R}}}$$

$$f) A = \frac{abc}{4r} \quad | \cdot r$$

$$A \cdot r = \frac{abc}{4} \quad | : A$$

$$\underline{\underline{r = \frac{abc}{4A}}}$$